AC/04.08.2018/RS1

SIES College of Arts, Science and Commerce
(Autonomous)
Affiliated to Mumbai University

Syllabus under Autonomy - July 2018

Program: F.Y.B.Sc.
Course: MATHEMATICS

Choice Based Credit System CBCS

with effect from the academic year 2018-19

Aim:

To attract mathematically able students and to provide for them an academically coherent undergraduate program, with courses that range from the fundamental to the advanced.

Broad Objectives:

The course; divided into two semesters, two papers in each semester, has the following goals for its learners:

1. To develop critical thinking, reasoning and logical skills
2. To improve analytical skills and its application to problem solving.
3. To take the learners from simple to difficult and from concrete to abstract
4. Have a deeper understanding of abstract mathematical theory and concepts
5. To improve capacity to communicate mathematical/logical ideas in writing

Course objectives: Paper1

On completion of this course of paper1 successfully, student should be able to

1. Understand order relation in IR and compute supremum and infimum of a subset of IR
2. State domain and range of standard functions and plot their graphs
3. Check convergence of a sequence and series of real numbers using tests and definition

Course objectives: Paper2

On completion of this course of paper 2 successfully, student should be able to

1. Compute gcd using Euclidean Algorithm and also compute last digit of powers of integers using congruences.
2. Write Mathematical proofs effectively.
3. Check for bijectivity of a function, write partitions w.r.t. equivalence relations.
4. Find roots of polynomials and gcd of polynomials.

SEMESTER I				
CALCULUS I **THEORY**				
Course Code	UNIT	TOPICS	Credits	L or P/week
SIUSMAT11	I	Real Number System, real valued functions of one variable and graphs	2	3L
	II	Sequences		
	III	Series		
ALGEBRA I				
Course Code	UNIT	TOPICS	Credits	L or P/week
SIUSMAT12	I	Integers and divisibility	2	3L
	II	Functions and Equivalence relations		
	III	Polynomials		
PRACTICALS				
Course Code	TOPICS		Credits	L or P/week
SIUSMATP1	Practicals based on courses SIUSMAT11 & SIUSMAT12		2	1P(=2L)

Teaching Pattern

1. Three lectures per week per course.
2. One Practical per week per batch per course.
3. Minimum 4 practicals to be conducted per batch per course in each semester.

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

SEMESTER I: PAPER I: CALCULUS I

Pre-requisites: Introduction to sets $\mathbb{N}, \mathbb{Q}, \mathbb{Z}, \mathbb{R}$

Unit I — Real Number System, real valued functions of one variable and graphs — 15 Lectures

- Real Numbers:Real number system and order properties of $\mathbb{R}$, Law of Trichotomy , Absolute value properties, AM-GM inequality, Cauchy-Schwarz inequality
- Intervals and neighbourhoods, Hausdorff property
- Bounded sets, Supremum and infimum, Maximum and minimum, Basic results: Continuum property (l.u.b.Axiom-statement) and consequences,Archimedean property and its applications.
- Functions and Graphs: Domain, Range, Types of function: Injective, Surjective, Bijective,Inverse, Composite.Graph of a function.Examples :constant function, identity function, absolute value, step function, floor and ceiling functions, trigonometric functions, linear and quadratic functions and their graphs. Graphs of functions such asx^3,$\frac{1}{x}$,$\frac{1}{x^2}$, $\sin\left(\frac{1}{x}\right)$, log(x) , a^x and e^x.Monotonic and strictly monotonic functions-definition and examples.

Unit II — Sequences — 15 Lectures

- Definition of a sequence and examples, Definition of convergent and divergent sequences.Limit of sequence, uniqueness of limit if it exists. Simple examples such as seq(1/n) , where convergence is checked using ϵ- n_0 definition.
- Sandwich theorem, Algebra of convergent sequences , Examples.
- Subsequences: Definition, Subsequence of a convergent sequence isconvergent and converges to the same limit.
- Cauchy sequence: Definition, every convergent sequence is a Cauchy sequence and conversely. Examples of Cauchy sequences.
- Monotonic and Bounded sequences: Definition of bounded sequence.Every convergent sequence is bounded .Monotone sequences and Monotone convergence theorem. Examples .

Unit III — Series — 15 Lectures

- Definition of Series as a Sequence of partial sums, Summation of a series, simple examples like Geometric series
- Convergent and divergent series, Necessary condition for convergence of series, Converse not true. Algebra of convergent series.
- Cauchy criterion for convergence of a series.
- Alternating series, Leibnitz Test, Examples.
- Absolute convergence implies convergence but not conversely. Conditional convergence.
- Convergence of a p- series
- Tests for convergence (Statements only):
    - Comparison test, limit form of comparison test, examples,

- Ratio test, examples,
- Root test, examples

SIUSMATP1 Practicals

1. Application based examples of Archimedean property, intervals, neighbourhood. Consequences of continuum property, infimum and supremum of sets.
2. Calculating limits of sequences using definition. Cauchy sequence, monotone sequence.
3. Sum of the series, conditional convergence, Leibnitz test. Tests of convergence, Examples.
4. Miscellaneous Theoretical questions based on full paper.

SEMESTER I: PAPER II: ALGEBRA I

Pre-requisites: Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, Relations, Permutations and combinations.

Unit I — Integers and divisibility — 15 Lectures

- Concepts of Statements, Propositions and Theorems, Logical Connectives and Truth Tables, Methods of proofs with basic examples.
- Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second), Binomial theorem for non-negative integer exponents, Pascal's rule.
- Divisibility in integers, division algorithm, Primes, Euclid's lemma, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers and that the g.c.d. can be expressed as $ma + nb, m, n \in \mathbb{Z}$, Euclidean algorithm, statement of Fundamental theorem of arithmetic, The set of primes is infinite.
- Congruences, definition and elementary properties, Euler'sϕ-function, Statements of Euler's theorem, Fermat's theorem and Wilson theorem, Applications.

Unit II — Functions and Equivalence relations — 15 Lectures

- Definition of a function, domain, co-domain and range of a function, composite functions, examples, Direct image $f(A)$ and inverse image $f^{-1}(B)$ for a function f; Injective, surjective, bijective functions; Composite of injective, surjective, bijective functions when defined; invertible functions, bijective functions are invertible and conversely; examples of functions including constant, identity, projection, inclusion;
- Binary operation as a function, properties, examples.
- Equivalence relation, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa. Congruence is an equivalence relation on $\mathbb{Z}$, Residue classes and partition of $\mathbb{Z}$.

Unit III — Polynomials — 15 Lectures

- Definition of a polynomial, polynomials over the field F where F = $\mathbb{Q}$, $\mathbb{R}$ or $\mathbb{C}$, Algebra of polynomials, degree of polynomial, basic properties,
- Division algorithm in F[X] (without proof), and g.c.d. of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem, A polynomial of degree n has at

most n roots , Complex roots of a polynomial in R[X] occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree n in C[X] has exactly n complex roots counted with multiplicity, A non constant polynomial in R[X] can be expressed as a product of linear and quadratic factors in R[X], necessary condition for a rational number p/q to be a root of a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is an irrational number where p is a prime number

SIUSMATP1 Practicals

1 Mathematical induction, Division Algorithm and Euclidean algorithm in $\mathbb{Z}$, primes and the Fundamental Theorem of Arithmetic.Congruences and Euler's ϕ-function, Fermat's little theorem, Euler's theorem and Wilson's theorem.
2 Functions (direct image and inverse image), Injective, surjective, bijective functions, finding inverses of bijective functions. Equivalence relations and Partitions.
3 Division Algorithm and GCD of polynomials. Factor Theorem, relation between roots and coefficients of polynomials, factorization.
4 Miscellaneous Theoretical questions based on full paper.

References for Paper I

1. Ajitkumar, S. Kumaresan.(2014). *A Basic Course in Real Analysis*, CRC press
2. R. R. Goldberg.(1964). *Methods of Real Analysis*, Oxford and IBH
3. James Stewart, Third Edition.(1994). *Calculus*, Brooks/Cole Publishing Company
4. T. M. Apostol.(1991). *Calculus Vol I*, Wiley & Sons (Asia) Pte. Ltd.
5. Ghorpade Limaye. (2000). *A Course in Calculus and Real Analysis*, Springer International Ltd
6. Binmore. (1982). *Mathematical Analysis*, Cambridge University Press
7. G. B. Thomas.(2009).*Calculus*, Addison Wesley

References for Paper II

1. Larry J. Gerstein.(2012). *Introduction to Mathematical Structures and Proofs,* Springer-Verlag, New York
2. J. P. Tremblay & R. Manohar.(1974).*Discrete Mathematical Structures with Applications to Computer Science,* McGraw Hill
3. David M. Burton.(2015). *Elementary Number Theory,* McGraw Hill Education (India) Private Ltd.
4. Norman L. Biggs.(1989).*Discrete Mathematics*, Clarendon Press, Oxford
5. I. Niven and S. Zuckerman.(1972).*Introduction to the theory of numbers,* Wiley Eastern
6. G. Birkoff and S. Maclane. (1965).*A Survey of Modern Algebra,*Mac Millan, New York
7. N. S. Gopalkrishnan.(2013).*University Algebra*, Ne Age International Ltd
8. I .N. Herstein, , John Wiley.(2006).*Topics in Algebra*
9. P. B. Bhattacharya S. K. Jain and S. R. Nagpaul.(1994). *Basic Abstract Algebra*, New Age International
10. Kenneth Rosen.(1999). *Discrete Mathematics and its applications*, Mc-Graw Hill International Edition, Mathematics Series.
11. Ajit Kumar & S Kumaresan.(2018).*Foundation Course in Mathematics,* Narosa

SEMESTER II

Course objectives: Paper1

On completion of this course of paper1 successfully, student should be able to

1. Test existence of limit and continuity and differentiability of a function at a point and on a set, identify types of discontinuity

2. Understand the relation between Continuity and Differentiability

3. Compute higher order derivatives

4. Understand the application of Mean Value theorems

5. Compute Local and Global Extremas of functions

Course objectives: Paper2

On completion of this course of paper 2 successfully, student should be able to

1. Solve system of linear equations using matrices.

2. Identify Vector spaces and their subspaces.

3. Write a Basis of given vector space and identify linear transformations.

SEMESTER II				
CALCULUS II THEORY				
Course Code	UNIT	TOPICS	Credits	L or P/week
SIUSMAT21	I	Limits and Continuous functions	2	3L
	II	Differentiation		
	III	Applications of derivatives		
ALGEBRA II				
Course Code	UNIT	TOPICS	Credits	L or P/week
SIUSMAT21	I	Systems of Linear Equations and Matrices	2	3L
	II	Vector spaces		
	III	Basis and Linear Transformation		
PRACTICALS				
Course Code	TOPICS		Credits	L or P/week
SIUSMATP2	Practicals based on courses SIUSMAT21 & SIUSMAT22		2	1P(=2L)

Teaching Pattern

1. Three lectures per week per course.
2. One Practical per week per batch per course.
3. Minimum 4 practicals to be conducted per batch per course in each semester.

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

SEMESTER II: PAPER I: CALCULUS II

Pre-requisites: Concepts of limits and continuity from classes XI and XII

Unit I Limits and Continuity of functions 15 Lectures

- ε - δ definition of limit of a function,
  Evaluation of limit of simple functions using the definition,
  Uniqueness of limit if it exists, Algebra of limits (with proof),
  Limit of composite function
- Sandwich theorem, Left hand, Right hand limits,
  non-existence of limits
- Infinite limits and Limits at infinity.
- Indeterminate forms, L'hospital rule without proof,
  examples of indeterminate forms
- Continuity of a real valued function at a point in terms of Limits,
  Continuity of a real valued function on a set in terms of Limits,
  examples, Continuity of a real valued function at end points of domain
- Sequential continuity
- Algebra of continuous functions
- Discontinuous functions, examples of removable and essential discontinuity, Continuous extension of a function
- Statements of properties of continuous functions on closed intervals such as Intermediate Value Property, Attainment of bounds, Location of roots and examples.

Unit II Differentiation 15 Lectures

- Definition of Derivatives of a real valued function of one variableat a point and on an open set
- Left / Right Derivatives
  Examples of differentiable and non differentiable functions
- Geometric/ Physical Interpretation of derivative
- Differentiable functions are continuous but not conversely
- Algebra of differentiable functions,
- Derivative of inverse functions, Implicit differentiation
  (only examples).
- Higher order derivatives
  Derivative of a composite function - Chain rule
  Higher order derivatives of some standard functions,
  Leibnitz rule.

Unit III Applications of derivatives 15 Lectures

- The Mean Value Theorems:
  Rolle's theorem
  Lagrange's mean value theorem. examples and applications
  Cauchy's mean value theorems, examples and applications
  Taylor's theorem with Lagrange's form of remainder
  Taylor's polynomial and applications.
- Applications of first and second derivatives:

Monotone increasing and decreasing function, examples,
Concave, Convex functions, Points of Inflection
Asymptotes, Definition of local maximum and local minimum, First derivative test for extrema, Necessary condition for extrema, Stationary points
Second derivative test for extrema, examples
Global maxima and minima

- Graphing of functions using first and second derivatives

SIUSMATP2 Practicals

1. Limits of functions and continuity. Properties of continuous functions.
2. Differentiability. Higher order derivatives, Leibnitz theorem.
3. Extreme values, increasing and decreasing functions. Mean value theorems and its applications.
4. Miscellaneous Theoretical questions based on full paper.

SEMESTER II: ALGEBRA II

Pre-requisites:	Review of vectors in $\mathbb{R}^2, \mathbb{R}^3$as points, Addition and scalar multiplication of vectors, Dot product, Scalar triple product, Length (norm) of a vector.

Unit I System of Linear equations and Matrices — 15 Lectures

- Parametric equation of lines and planes, homogeneous and non-homogeneous system of linear equations, the solution of homogeneous system of m linear equations in n unknowns by elimination and their geometrical interpretation for $(n, m) = (1,2), (1,3), (2,2), (2,3), (3,3)$; definition of $n-$tuples of real numbers, sum of two $n-$tuples and scalar multiple of an $n-$tuple.
- Matrices with real entries; addition, scalar multiplication and multiplication of matrices; transpose of a matrix, types of matrices: zero matrix, identity matrix, scalar matrices, diagonal matrices, upper triangular matrices, lower triangular matrices, symmetric matrices, skew-symmetric matrices, Invertible matrices; identities such as $(AB)^t = B^t A^t$, $(AB)^{-1} = B^{-1}A^{-1}$.
- System of linear equations in matrix form, elementary row operations, row echelon matrix, Gaussian elimination method, to deduce that the homogeneous system of m linear equations in n unknowns has a non-trivial solution if $m < n$.

Unit II Vector spaces — 15 Lectures

- Definition of a real vector space, examples such as $\mathbb{R}^n, \mathbb{R}[x], M_{m\times n}(\mathbb{R})$, space of all real valued functions on a non empty set.
- Subspace: definition, examples: lines, planes passing through origin as subspaces of $\mathbb{R}^2, \mathbb{R}^3$, upper triangular matrices, diagonal matrices, symmetric matrices, skew-symmetric matrices as subspaces of $M_n(\mathbb{R})$ $(n = 2, 3)$; $P_n(x) = \{a_0 + a_1x + \cdots + a_nx^n \mid a_i \in \mathbb{R}\ \forall\ 0 \leq i \leq n\}$ as a subspace $\mathbb{R}[x]$, the space of all solutions of a homogeneous system of m linear equations in n unknowns as a subspace of $\mathbb{R}^n$. Properties of a subspace such as necessary and sufficient condition fora non empty subset to be a subspace of a vector space, arbitrary intersection of subspaces of a vector space is a subspace, union of two subspaces is a subspace if and only if one is a subset of the other.
- Finite linear combinations of vectors in a vector space; the linear span $L(S)$ of a non-empty subset S of a vector space, S is a generating set for $L(S)$, $L(S)$ is a vector subspace of$V$
- Linearly independent/linearly dependent subsets of a vector space, a subset $\{v_1, v_2, \ldots, v_k\}$ of a vector space is linearly dependent if and only if $\exists\ i \in \{1, 2, \cdots, k\}$ such that v_i is a linear combination of the othervectors v_j s.

Unit III Basis and Linear Transformations — 15 Lectures

- Basis of a vector space, dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, minimal generating subset of a vector space is a basis of a vector space, any two basis of a vector space have the same number of elements, any set of n linearly independent vectors in an n−dimensional vector space is a basis, any collection of n+1 linearly independent vectors in an n−dimensional vector space is linearly dependent.

- If W_1, W_2 are two subspaces of a vector space V then $W_1 + W_2$is a subspace of the vector space V of dimension $\dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$, extending any basis of a subspace W of a vector space V to a basis of the vector space V.
- Linear transformations; kernel and image, matrix associated with a linear transformation, properties such as: for a linear transformation T, kernel(T) is a subspace of the domain space of T and the image image(T) is a subspace of the co-domain space of T. If V, W are real vector spaces with $\{v_1, v_2, \cdots, v_n\}$ a basis of V and $w_1, w_2, \cdots, w_n$ any vectors in W then there exists a unique linear transformation $T : V \to W$ such that $\mathrm{T}(v_j) = w_j \ \ \forall\, 1 \le j \le n$, Rank nullity theorem (statement only) and examples.

SIUSMATP2	Practicals
1	Finding parametric equations and row echelon form. Solving system Ax=b by Gauss elimination
2	Proving the given sets to be subspaces, Linear span of a non-empty subset of a vector space, linear independence/dependence
3	Finding basis of a vector spaces. Verifying whether a set is a basis of a vector space. Extending basis of a subspace to a basis of a finite dimensional vector space. Verifying whether a map $T : X \to Y$ is a linear transformation, finding kernel of a linear transformation and matrix associated with a linear transformation, verifying the Rank Nullity theorem.
4	Miscellaneous Theoretical questions based on full paper.

References for Paper I

1. Ajitkumar, S. Kumaresan.(2014).*A Basic Course in Real Analysis,* CRC press
2. R. R. Goldberg. (1964).*Methods of Real Analysis*, Oxford and IBH
3. James Stewart.(1994).*Calculus*, Third Edition, Brooks/cole Publishing Company
4. T. M. Apostol.(1991)*. Calculus Vol I*, Wiley & Sons (Asia) Pte. Ltd.
5. Ghorpade&Limaye .(2000). *A Course in Calculus and Real Analysis*, Springer International Ltd
6. Binmore.(1982).*Mathematical Analysis*, Cambridge University Press
7. G. B. Thomas.(2009). *Calculus,* 12th Edition

References for Paper II

1. Serge Lang. (1986). *Introduction to Linear Algebra*, Second Edition, Springer.
2. S. Kumaresan.(2000).*Linear Algebra, A Geometric Approach*, Prentice Hall of India Pvt. Ltd
3. M. Artin.(1991).*Algebra*, Prentice Hall of India Private Limited
4. K. Hoffman and R. Kunze.(1971). *Linear Algebra*, Tata McGraw-Hill, New Delhi.
5. Gilbert Strang.(1900).*Linear Algebra and its applications*, International Student Edition.

SCHEME OF EVALUATION

1. Semester End Theory Examination:

The performance of the learners shall be evaluated into two parts. The learner's performance shall be assessed by Internal Assessment with 40% marks in the first part; and by conducting the Semester End Examinations with 60% marks in the second part. The allocation of marks for the Internal Assessment and Semester End Examinations are as shown below:

(a) Internal assessment 40% :

Sr No	Evaluation type	Marks
1	One class test	20
2	Viva	10
3	Assignment/Project/Presentation	10
	Total	40

(b) External Theory examination 60 % :

Duration – 2 hours.
Question Paper Pattern:- Four questions each of 15 marks.
One question on each unit (Questions 1, 2, 3).
Question 4 will be based on entire syllabus.
All questions shall be compulsory with not more than 50% internal choice within the questions. Question may be subdivided into sub-questions a, b, c.

2. Semester End Practical Examination:

At the end of semesters I & II, practical examination of 2 hours duration and 100 marks shall be conducted for the courses SIUSMATP1 and SIUSMATP2.

Sr. No.	Evaluation type	Marks
1	Part A: Questions from SIUSMAT11	40
2	Part B: Questions from SIUSMAT12	40
3	Journal	10
4	Class Work	10

Guidelines about conduct of Practicals
The practical session should consist of discussion between the teacher and the students in which students should participate actively. Each practical of every course of semesters I & II shall contain ten problems out of which minimum five have to be written in the journal. A student must have a certified journal to appear for the practical examination.