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SIES College of Arts, Science & Commerce

RISE WITH EDUCATION

NAAC REACCREDITED "A" GRADE, CGPA 3.51/4.00

(AUTONOMOUS)

SIES College of Arts, Science and Commerce

(Autonomous)

(Affiliated to University of Mumbai)

Programme: B.Sc.

Subject: Mathematics

Class: S.Y. B.Sc. Semester III & IV(CBCS)

Syllabus Revised in June 2022

Choice Based Credit System (CBCS)

with effect from the academic year 2022-23

CONTENTS

1. Preamble

Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematics to real world problems has increased by leaps and bounds. It is imperative that the content of the undergraduate syllabi of Mathematics should support other branches of science such as Physics, Chemistry, Statistics, Computer Science, Biotechnology. This syllabus of S.Y.B.Sc. Mathematics has been designed to provide learners sufficient knowledge and skills enabling them to undertake further studies in mathematics and its allied areas. There are 6 theory and 2 practical courses spread over semesters III and IV covering the core areas of Mathematics such as Linear Algebra, Real Analysis, Multivariable Calculus, Differential Equations and Numerical Methods.

2. Learning Objectives

- To develop critical thinking, reasoning and logical skills of the learners
- To improve learners' analytical and problem solving skills
- To take the learners from simple to difficult and from concrete to abstract
- To equip learners with a deeper understanding of abstract theory and concepts
- To improve learners' capacity to communicate mathematical/logical ideas in writing.

3. Programme Outcomes

SIES has integrated the Learning Outcome Based Curriculum Framework in the syllabi of all the programmes since the academic year 2021-22. Upon completing the B.Sc. Mathematics Programme, the students are expected to develop the following abilities and skills:

I. **Solving Complex Problems:**
Applying the knowledge of various courses learned under a program with an ability to break down complex problems into simple components, by designing processes required for problem solving.

II. **Critical Thinking and reasoning ability:**
Exhibits ability to understand abstract concepts,analyze, and apply them in problem solving. Ability to formulate and develop logical arguments. developing the ability to think with different perspectives and ideas. (Skills necessary for progression to higher education and research.)

III. **Research Aptitude:**
Acquiring the ability to explore and gain knowledge in independent ways through reading assignments, problem solving assignments, projects, seminars, presentations.

IV. **Information and Digital literacy**:
Equip to select, apply appropriate tools and techniques, resources through electronic media for the purpose of visualizing mathematical objects, geometrical interpretations, coding, and analyzing data.

V. **Sound Disciplinary knowledge:**
Demonstrate comprehensive knowledge and understanding of the fundamental concepts and theories of mathematics; apply them to interdisciplinary areas of study.

VI. **Communicating Mathematical Ideas:**
Organize and deliver mathematical ideas through effective written, verbal, graphical/virtual communications.

4. Course structure with minimum credits and Lectures/ Week

SEMESTER III				
INTEGRAL CALCULUS OF ONE VARIABLE				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT31	I	Infinite Series	2	3
	II	Riemann Integration and applications		
	III	Indefinite and improper integrals		
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT32	I	System of Equations and Matrices	2	3
	II	Vector Spaces over IR		
	III	Determinants, Linear Equations (Revisited)		
NUMERICAL METHODS				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT33	I	Solutions of algebraic and transcendental equations	2	3
	II	Interpolation, Curve fitting, Numerical integration		
	III	Solutions of linear system of Equations and Numerical Differentiation		
PRACTICALS				
SIUSMATP3	I	Practicals based on **SIUSMAT31, SIUSMAT32, SIUSMAT33**	3	6
SEMESTER IV				
MULTIVARIABLE DIFFERENTIAL CALCULUS				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT41	I	Functions of several variables	2	3
	II	Differentiation of Scalar Fields		
	III	Applications of Differentiation of Scalar Fields and Differentiation of Vector Fields		
Linear Algebra II				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT42	I	Linear transformation, Isomorphism, Matrix associated with L.T.	2	3
	II	Inner product spaces		
	III	Eigenvalues, Eigen vectors, diagonalizable matrix		
ORDINARY DIFFERENTIAL EQUATIONS				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT43	I	Higher Order linear Differential Equations	2	3
	II	Systems of First Order Linear differential equations		
	III	Numerical Solutions of Ordinary Differential Equations		
PRACTICALS				
SIUSMATP4		Practicals based on Courses **SIUSMAT41, SIUSMAT42, SIUSMAT43**	3	6

5. Consolidated Syllabus for semester III & IV with Course Outcomes

SEMESTER III

Course Code: SIUSMAT31
Course Name: INTEGRAL CALCULUS OF ONE VARIABLE

Expected Course Outcomes:

On completion of this course, students are expected to

1. State the definitions and prove results based on concepts summation and convergence of a series, the lower and upper Riemann integrals, the beta, gamma functions, indefinite and improper integrals.
2. Apply various definitions and results learnt to solve problems on convergence of infinite series, improper integrals, upper and lower sums and checking integrability, problems in physics
3. Test the validity of mathematical statements and converses based upon the gained knowledge, choose appropriate methods to discuss integrability of a function, convergence of an integral and that of a series.

SIUSMAT31: INTEGRAL CALCULUS OF ONE VARIABLE

Unit I: Infinite Series (15 Lectures)

1.1 Definition of Series as a Sequence of partial sums, Summation of a series, simple examples like Geometric series. Convergent and divergent series, Necessary condition for convergence of series, Converse not true. Algebra of convergent series. Cauchy criterion for convergence of a series.

1.2 Alternating series, Leibnitz Test, Examples. Absolutely convergent series. Absolute convergence implies convergence but not conversely. Conditional convergence.

1.3. Convergence of a p- series- $\sum_{n=0}^{\infty} \frac{1}{n^p}$; divergence of the Harmonic series $\sum_{n=0}^{\infty} \frac{1}{n}$

1.4 Tests for convergence of an infinite series: Comparison test, limit form of comparison test, Ratio test, Limit form of ratio test, Root test, Limit form of root test.

Unit II: Riemann Integration and applications (15 Lectures)

2.1 Idea of approximating the area under a curve by inscribed and circumscribed rectangles. Partitions of an interval. Refinement of a partition. Upper and Lower sums for a bounded real valued function on a closed and bounded interval. Riemann integrability and the Riemann integral.

2.2 Criterion for Riemann integrability. Characterization of the Riemann integral as the limit of a sum. Examples.

2.3 Algebra of integrable functions: Sum, scalar multiplication, product of integrable functions are integrable. Properties of integrable functions: Integral of a non negative function is nonnegative, $\left|\int_a^b f\right| \leq \int_a^b |f|$; domain additivity, Examples and counterexamples.

2.4 Riemann integrability of a continuous function, and more generally of a bounded function whose set of discontinuities has only finitely many points. Riemann integrability of monotone functions.

Unit III: Indefinite and improper integrals (15 lectures)

1.1 Definition of Indefinite Riemann Integral function and its continuity.
1.2 Mean Value Theorem of Integral Calculus, 1^{st} and 2^{nd} Fundamental theorems of Integral Calculus, Integration by parts, Change of Variable formula for integration.
1.3 Improper integrals of types I & II, Absolute convergence, convergence of some special integrals, comparison test.
1.4 Definition and properties of beta and gamma functions. Relationship between Beta and gamma functions (statement and examples).

Topics for assignment and self-study:

i) Applications of definite Integrals: Area between curves, finding volumes of solids of revolution Lengths of plane curves, Areas of surfaces of revolution, finding average value of a function, finding mass and center of mass, any other application.
ii) Abel's and Dirichlet's Test of convergence of an infinite series, Applications of Infinite series.

Reference Books:

1) Howard Anton, Calculus-A new Horizon, Sixth Edition, John Wiley and Sons Inc.
2) K. Stewart, Calculus, Booke /Cole Publishing Co.
3) Bartle and Sherbet, Introduction to real analysis, fourth edition or earlier, John Wiley and Sons Inc.
4) Ajit Kumar, Kumaresan S., A Basic Course in Real Analysis, first edition, CRC Press.
5) Apostol T. , Calculus Vol.2, any edition, John Wiley and Sons Inc.
6) J .E. Marsden, A. J. Tromba and A. Weinstein, Basic multivariable calculus, 3rd edition, W.H. Freeman and Co Ltd.

Course Code: SIUSMAT32
Course Name: Linear Algebra I

Expected Course Outcomes:

On completion of this course, students are expected to

1. State the definitions and prove the results of Systems of homogeneous and non-homogeneous linear equations, row echelon form of matrices, elementary matrices, Vector space over R, its basis, determinant.
2. Solve problems in system of linear equations using Gaussian elimination, Cramer's rule, *LU* Decomposition, finding inverse of matrix, checking Linear independence of subsets of a vector space

SIUSMAT32: LINEAR ALGEBRA I

Note: Revision of relevant concepts is necessary.

Unit I. System of Equations, Matrices (15 Lectures)

1. Systems of homogeneous and non-homogeneous linear equations, Simple examples of finding solutions of such systems. Geometric and algebraic understanding of the solutions. Matrices (with real entries), Matrix representation of systems of homogeneous and non-homogeneous linear equations. Algebra of solutions of systems of homogeneous linear equations. A system of homogeneous linear equations with a number of unknowns more than the number of equations has infinitely many solutions.
2. Elementary row and column operations. Row equivalent matrices. Row reduction (of a matrix to its row echelon form). Gaussian elimination. Applications to solving systems of linear equations. Examples.
3. Elementary matrices. Relation of elementary row operations with elementary matrices. Invertibility of elementary matrices. Consequences such as (i) a square matrix is invertible if and only if its row echelon form is invertible. (ii) invertible matrices are products of elementary matrices. Examples of the computation of the inverse of a matrix using Gauss elimination method.

Unit II. Vector space over R (15 Lectures)

1. Definition of a vector space over R. Subspaces; criterion for a nonempty subset to be a subspace of a vector space. Examples of vector spaces, including the Euclidean space R^n, lines, planes and hyperplanes in R^n passing through the origin, space of systems of homogeneous linear equations, space of polynomials, space of various types of matrices, space of real valued functions on a set.
2. Intersections and sums of subspaces. Direct sums of vector spaces. Quotient space of a vector space by its subspace.
3. Linear combination of vectors. Linear span of a subset of a vector space. Definition of a finitely generated vector space. Linear dependence and independence of subsets of a vector space.
4. Basis of a vector space. Basic results that any two bases of a finitely generated vector space have the same number of elements. Dimension of a vector space. Examples. Bases of a vector space as a maximal linearly independent sets and as minimal generating sets.

Unit III. Determinants, Linear Equations (Revisited) (15 Lectures)

1. Inductive definition of the determinant of a $n \times n$ matrix (e. g. in terms of expansion along the first row). Example of a lower triangular matrix. Laplace expansions along an arbitrary row or column. Determinant expansions using permutations
2. Basic properties of determinants (Statements only);
   (i) det A = det A^T. (ii) Multilinearity and alternating property for columns and rows.
   (iii) A square matrix A is invertible if and only if det $A \neq 0$.
   (iv) Minors and cofactors. Formula for A^{-1} when det $A \neq 0$. (v) det(AB) = det A det B.
3. Row space and the column space of a matrix as examples of vector space. Notion of row rank and the column rank. Equivalence of the row rank and the column rank. Invariance of rank upon elementary row or column operations. Examples of computing the rank using row reduction.
4. Relation between the solutions of a system of non-homogeneous linear equations and the associated system of homogeneous linear equations. Necessary and sufficient condition for a system of non-homogeneous linear equations to have a solution (viz., the rank of the coefficient matrix equals the rank of the augmented matrix [A|B]). Equivalence of statements (in which A denotes an $n \times n$ matrix) such as the following:
   (i) The system $Ax = b$ of non-homogeneous linear equations has a unique solution.
   (ii) The system $Ax = 0$ of homogeneous linear equations has no nontrivial solution.
   (iii) A is invertible. (iv) det $A \neq 0$. (v) rank(A) = n.
5. Cramer's Rule. *LU and PLU* Decomposition. If a square matrix A is a matrix that can be reduced to row echelon form U by Gauss elimination without row interchanges, then A can be factored as $A = LU$ where L is a lower triangular matrix.

Reference books

1) Howard Anton, Chris Rorres, Elementary Linear Algebra, Wiley Student Edition.
2) Serge Lang, Introduction to Linear Algebra, Springer.
3) S Kumaresan, Linear Algebra - A Geometric Approach, PHI Learning.
4) K. Hoffman and R. Kunze : Linear Algebra, Tata McGraw-Hill, New Delhi.

Course Code: SIUSMAT33
Course Name: Numerical Methods

Expected Course Outcomes:
On completion of this course, students are expected to

1. State definitions of concepts such as relative,absolute and percentage errors,accuracy,precision and explain Interpolation using different types of operators-Forward,backward and shift. State and derive numerical methods for various mathematical operations and tasks, such as interpolation, differentiation, integration, the solution of linear and nonlinear equations, and the solution of differential equations.
2. Apply numerical techniques to find the roots of nonlinear equations, solution of systems of linear equations,numerical integration and differentiation
3. Evaluate limitations,advantages,disadvantages and accuracy of different numerical methods

SIUSMAT33: Numerical Methods

Unit I. Solution of Algebraic and Transcendental Equations (15L)

1. Measures of Errors: Relative, absolute and percentage errors, Accuracy, and precision: Accuracy to n decimal places, accuracy to n significant digits or significant figures, Rounding and Chopping of a number, Types of Errors: Inherent error, Round-off error, and Truncation error.
2. Iteration methods based on the first degree equation: Bisection method, General Iteration method: Fixed point iteration method, Newton-Raphson method. Secant method. Regula-Falsi method. Derivations and geometrical interpretation and rate of convergence of all above methods to be covered.

Unit II. Interpolation, Curve fitting, Numerical Integration(15L)

1. Interpolation: Lagrange's Interpolation. Finite difference operators: Forward Difference operator, Backward Difference operator, Shift operator, Newton's forward difference interpolation formula, Newton's backward difference interpolation formula. Derivations of all above methods.
2. Curve fitting: linear curve fitting. Quadratic curve fitting.
3. Numerical Integration: Trapezoidal Rule. Simpson's $\frac{1}{3}$rd Rule. Simpson's $\frac{3}{8}$th Rule. Derivations of all the above three rules to be covered.

Unit III. Solution Linear Systems of Equations,Numerical Differentiation(15L)

1. Linear Systems of Equations: LU Decomposition Method (Dolittle's Method and Crout's Method). Gauss-Seidel Iterative method, Gauss-Jacobi method.
2. Numerical Differentiation: Introduction,Derivatives using Newton's forward and backward interpolation formula,maxima and minima..

Reference Books:

1. Kendall E. and Atkinson; An Introduction to Numerical Analysis; Wiley.
2. M. K. Jain, S. R. K. Iyengar and R. K. Jain; Numerical Methods for Scientific and Engineering Computation; New Age International Publications.
3. S. Sastry; Introductory methods of Numerical Analysis; PHI Learning.
4. An introduction to Scilab-Cse iitb

Additional Reference Books:

1. S.D. Comte and Carl de Boor; Elementary Numerical Analysis, An algorithmic approach; McGraw Hill International Book Company.
2. Hildebrand F.B.; Introduction to Numerical Analysis; Dover Publication, NY.
3. Scarborough James B.; Numerical Mathematical Analysis; Oxford University Press, New Delhi.

Course Code: SIUSMATP3

Course Name: Practicals based on SIUSMAT31, SIUSMAT32, SIUSMAT33

Expected Course Outcomes

Upon completion of this course, students are expected to

1. Apply various definitions, results and methods learnt in three theory courses to plot graphs and solve problems.
2. Explore mathematical softwares/mobile apps like Matlab/ Scilab/ Geogebra/ SAGE/ Desmos to solve problems and visualize solids. (free and open versions)
3. Test validity of mathematical statements using results and constructing appropriate examples

Practicals in SIUSMAT31

1. Examples of convergent / divergent series and algebra of convergent series.
2. Problems based on Tests for convergence of series.
3. Calculation of upper sum, lower sum and Riemann integral.
4. Problems on properties of Riemann integral.
5. Problems on fundamental theorem of calculus, mean value theorems, integration by parts.
6. Convergence of improper integrals, tests for convergence. Properties of Beta, Gamma Functions.
7. Miscellaneous Questions based on all units.

Practicals in SIUSMAT32

1. Systems of homogeneous and non-homogeneous linear equations.
2. Elementary row/column operations and Elementary matrices.
3. Vector spaces, Subspaces.
4. Linear Dependence/independence, Basis, Dimension.
5. Determinant and Rank of a matrix.
6. Solution to a system of linear equations, LU decomposition
7. Miscellaneous Questions based on all units.

Practicals in SIUSMAT33

Use of scientific calculators/free open source mathematical softwares is encouraged.

1. Errors,Approximations,rounding
2. Roots of transcendental equations using Newton-Raphson method, Secant method. Regula-Falsi method, Iteration Method.
3. Interpolating polynomial by Lagrange's Interpolation, Newton forward and backward difference Interpolation.
4. Curve fitting, Trapezoidal Rule, Simpson's $\frac{1}{3}$ rd Rule, Simpson's $\frac{3}{8}$th Rule.
5. LU decomposition method, Gauss-Seidel and Gauss Jacobi Iterative method.
6. Finding first order,second order derivatives,maxima,minima using Newton's forward and backward difference formula.
7. Miscellaneous theory questions based on all units.

SEMESTER IV

Course Code: SIUSMAT41
Course Name: Multivariable Differential Calculus
Expected Course Outcomes:

On completion of this course, the students are expected to

1. State the definitions and prove results based on concepts continuity, partial and directional derivatives, the gradient vector, total derivative of scalar and vector fields.
2. Apply various definitions learnt to identify and plot quadric surfaces and level curves, compute gradient, partial and directional derivatives, Jacobian and total derivatives, extreme values.
3. Test the validity of mathematical statements and converses based upon the gained knowledge, to discuss differentiability of a function,existence of derivatives.

SIUSMAT41: Multivariable Differential Calculus
Unit I: Functions of several variables (15 Lectures)

1.1 The Euclidean inner product on R^n and Euclidean norm function on R^n, distance between two points. Review of vectors with special emphasis on R^2 and R^3. Real valued function of several variable ($R^n \rightarrow R$ scalar fields), examples, graph of a scalar field. Level sets (Level curves and surfaces, with examples in R^2 and R^3). Vector valued functions of several variables (from $R^n \rightarrow R^m$, vector fields), Component functions, examples.
1.2 Sequences, Limits and Continuity: Sequence in R^n[with emphasis on R^2 and R^3 and their limits. Neighborhoods in R^n. Limits and continuity of scalar fields. Composition of continuous functions. Sequential characterizations. Algebra of limits and continuity. Iterated limits. Limits and continuity of vector fields. Algebra of limits and continuity vector fields. (without proof).
1.3 Directional derivatives and partial derivatives of scalar fields: definition and examples, with emphasis on R^2 and R^3. Mean value theorem for derivatives of scalar fields.
1.4 Limits, Continuity of vector fields, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity and components of a vector field.

Unit II: Differentiation (15 Lectures)
2.1 Differentiability of scalar fields (in terms of linear transformation). The concept of (total) derivative. Uniqueness of total derivative of a differentiable function at a point. Examples of functions of two or three variables. Increment Theorem. Basic properties include (i) continuity at a point of differentiability, (ii) existence of partial derivatives at a point of differentiability, and (iii) differentiability when the partial derivatives exist and are continuous.
2.2 Gradient. Relation between total derivative and gradient of a function. Chain rule. Geometric properties of gradient. Tangent planes.
2.3 Euler’s Theorem.
2.4 Higher order partial derivatives. Mixed Partial derivatives Theorem (n=2).

Unit III: Applications (15 lectures)

3.1 The maximum and minimum rate of change of scalar fields. Average value of a scalar function
3.2 Hessian matrix, Second order Taylor's Theorem for twice continuously differentiable scalar fields. Examples. Method of Lagrange Multipliers.
3.3 Maxima, minima and saddle points. Second derivative test for extrema of functions of two variables.
3.4 Differentiability of vector fields, Jacobian matrix, Relation between differentiability and Jacobian, differentiability of a vector field at a point implies continuity. The chain rule for derivative of a vector field (statements only)

Topics for assignment and self-study:

1. Using Geogebra or Desmos or any other suitable mathematical software to plot quadric surfaces.
2. Demonstrating any mathematical software through an application.
3. Applications of solid geometry in other fields of study

Reference Books:

1. T. Apostol: Calculus, Vol.2, John Wiley.
2. J. Stewart, Calculus, Brooke/Cole Publishing Co.
3. G.B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, Addison-Wesley.
4. Sudhir R. Ghorpade and Balmohan V. Limaye, A Course in Multivariable Calculus and Analysis, Springer International Edition.
5. Howard Anton, Calculus-A new Horizon, Sixth Edition, John Wiley and Sons Inc.
6. Openstax Learning Resources: Calculus volume I, II and III, https://openstax.org/details/books/calculus-volume-3

Course Code: SIUSMAT42
Course Name: LINEAR ALGEBRA II
Expected Course Outcomes:

On completion of this course, students are expected to

1. State the definitions and prove the results in kernel and image of linear transformations, matrix associated with linear transformation, Inner Products and Orthogonality, Eigenvalues, Eigenvectors and Diagonalization.
2. Solve problems of finding kernel and image of linear transformation, finding matrix associated with linear transformation, finding orthonormal set using Gram-Schmidt orthogonalization, finding eigenvalues, eigenvectors and Diagonalizing a matrix.

SIUSMAT42: LINEAR ALGEBRA II

UNIT I. Linear Transformations

1. Definition of a linear transformation of vector spaces; elementary properties. Examples. Sums and scalar multiples of linear transformations. Composites of linear transformations. A Linear transformation of $V \to W$, where V, W are vector spaces over R with V a finite-dimensional vector space, is completely determined by its action on an ordered basis of V.
2. Null-space (kernel) and the image (range) of a linear transformation. Nullity and rank of a linear transformation. Rank-Nullity Theorem (Fundamental Theorem of Homomorphism).
3. Matrix associated with linear transformation of $V \to W$ where V and W are finite dimensional vector spaces over R. Matrix of the composite of two linear transformations. Invertible linear transformations (isomorphisms), Linear operator, Effect of change of bases on matrices of linear operator.
4. Equivalence of the rank of a matrix and the rank of the associated linear transformation. Similar matrices.

UNIT II. Inner Products and Orthogonality

1. Inner product spaces (over R). Examples, including the Euclidean space R^n and the space of real valued continuous functions on a closed and bounded interval. Norm associated with an inner product. Cauchy-Schwarz inequality. Triangle inequality.
2. Angle between two vectors. Orthogonality of vectors. Pythagoras theorem and some geometric applications in R^2. Orthogonal sets, Orthonormal sets. Gram-Schmidt orthogonalization process. Orthogonal basis and orthonormal basis for a finite-dimensional inner product space.
3. Orthogonal complement of any set of vectors in an inner product space. Orthogonal complement of a set is a vector subspace of the inner product space. Orthogonal decomposition of an inner product space with respect to its subspace. Orthogonal projection of a vector onto a line (one dimensional subspace). Orthogonal projection of an inner product space onto its subspace.

UNIT III. Eigenvalues, Eigenvectors and Diagonalization

1. Eigenvalues and eigenvectors of a linear transformation of a vector space into itself and of square matrices. The eigenvectors corresponding to distinct eigenvalues of a linear transformation are linearly independent. Eigenspaces. Algebraic and geometric multiplicity of an eigenvalue.
2. Characteristic polynomial. Properties of characteristic polynomials (only statements). Examples. Cayley-Hamilton Theorem. Applications.
3. Invariance of the characteristic polynomial and eigenvalues of similar matrices.
4. Diagonalizable matrix. A real square matrix A is diagonalizable if and only if there is a basis of R^n consisting of eigenvectors of A. (Statement only - $A_{n\times n}$ is diagonalizable if and only if sum of algebraic multiplicities is equal to sum of geometric multiplicities of all the eigenvalues of $A = n$). Procedure for diagonalizing a matrix.
5. Spectral Theorem for Real Symmetric Matrices (Statement only). Examples of orthogonal diagonalization of real symmetric matrices. Applications to quadratic forms and classification of conic sections.

Reference books

1. Howard Anton, Chris Rorres, Elementary Linear Algebra, Wiley Student Edition.
2. Serge Lang, Introduction to Linear Algebra, Springer.
3. S Kumaresan, Linear Algebra - A Geometric Approach, PHI Learning.
4. K. Hoffman and R. Kunze : Linear Algebra, Tata McGraw-Hill, New Delhi.

Course Code: SIUSMAT43
Course Name: ORDINARY DIFFERENTIAL EQUATIONS

Expected Course Outcomes

On completion of the course, students are expected

1. To have a working knowledge of basic application problems described by second order linear differential equations with constant coefficients.
2. To find the complete solution of a nonhomogeneous differential equation as a linear combination of the complementary function and a particular solution,by the method of undetermined coefficients and variation of parameters.
3. Create and analyze mathematical models using higher order differential equations to solve application problems.

SIUSMAT43: ORDINARY DIFFERENTIAL EQUATIONS

Unit I. Higher order Linear Differential equations (15 Lectures)

1. The general nth order linear differential equations, Linear independence, Existence and uniqueness theorem, Classification: homogeneous and non-homogeneous, Wronskian, General solution of homogeneous and non-homogeneous LDE, The Differential operator and its properties.

2. Higher order homogeneous linear differential equations with constant coefficients, the auxiliary equations, Roots of the auxiliary equations: real and distinct, real, and repeated, complex, and complex repeated.

3. Higher order homogeneous linear differential equations with constant coefficients, the method of undetermined coefficients, method of variation of parameters.

4. The inverse differential operator and particular integral, Evaluation of $\frac{1}{f(D)}$ for the functions like e^{ax} , $sin\ ax$, $cos\ ax$, x^m, $x^m sin\ ax$, $x^m cos\ ax$, $e^{ax}V$ and xV where V is any function of x.

5. Higher order linear differential equations with variable coefficients: The Cauchy's equation: $a_0x^3\frac{d^3y}{dx^3} + a_1x^2\frac{d^2y}{dx^2} + a_2x\frac{dy}{dx} + a_3y = f(x)\ where\ a_0, a_1, a_2, a_3 \in R$, Legendre's linear equation: $a_0(ax + b)^3\frac{d^3y}{dx^3} + a_1(ax + b)^2\frac{d^2y}{dx^2} + a_2(ax + b)\frac{dy}{dx} + a_3y = f(x)\ where\ a_0, a_1, a_2, a_3 \in R$,

Unit II. Systems of First Order Linear Differential Equations (15 Lectures)

1. Existence and uniqueness theorem for the solutions of initial value problems for a system of two first order linear differential equations in two unknown functions x, y of a single independent variable t, of the form: $\frac{dx}{dt}= F(t, x, y)$, $\frac{dy}{dt} = G(t, x, y)$ (Statement only).

2. Homogeneous linear system of two first order differential equations in two unknown functions of a single independent variable t of the form $\{\frac{dx}{dt} = a_1(t)x + b_1(t)y\ ,\ \frac{dy}{dt} = a_2(t)x + b_2(t)y\}$

3. Wronskian for a homogeneous linear system of first order linear differential equations in two functions x, y of a single independent variable t. Vanishing properties of the Wronskian. Relation with linear independence of solutions.

4. Homogeneous linear systems with constant coefficients in two unknown functions x, y of a single independent variable t. Auxiliary equation associated with a homogenous system of equations with constant coefficients. Description for the general solution depending on the roots and their multiplicities of the auxiliary equation, proof of independence of the solutions. Real form of solutions in case the auxiliary equation has complex roots.

5.Non-homogeneous linear system of two first order differential equations in two unknown functions of a single independent variable t of the form $\{\frac{dx}{dt} = a_1(t)x + b_1(t)y + f_1(t), \frac{dy}{dt} = a_2(t)x + b_2(t)y + f_2(t)\}$. General Solution of non-homogeneous systems. Relation between the solutions of a system of non-homogeneous linear differential equations and the associated system of homogeneous linear differential equations.

Unit III. Numerical Solution of Ordinary Differential Equations (15 lectures)

1. Numerical Solution of initial value problem of first order ordinary differential equation using:
(i) Picard's method for successive approximation and its convergence,
(ii) Taylor's series method,
(iii) Euler's method and error estimates for Euler's method
(iv) Modified Euler's Method,
(v) Runge-Kutta method of second order and its error estimates,
(vi) Runge-Kutta fourth order method.

2. Numerical solution of simultaneous and higher order ordinary differential equation using:
(i) Runge-Kutta fourth order method for solving simultaneous ordinary differential equation,
(ii) Finite difference method for the solution of two-point linear boundary value problem.

Reference Books

1. Units 5, 6, 7 and 8 of E.D. Rainville and P.E. Bedient; Elementary Differential Equations; Macmillan.
2. Units 5, 6 and 7 of M.D. Raisinghania; Ordinary and Partial Differential Equations; S. Chand.
3. G.F. Simmons; Differential Equations with Applications and Historical Notes; Taylor's and Francis.
4. Elementary Differential Equations and Boundary Value Problems; Boyce DiPrima; John Wiley & Sons (Asia) Pte Ltd
5. K. Atkinson, W.Han and D Stewart, Numerical Solution of Ordinary Differential Equations, Wiley.

Course Code: SIUSMATP4

Course Name: Practicals in SIUSMAT41, SIUSMAT42, SIUSMAT43

Expected Course Outcomes

Upon completion of this practical course, students are expected to

1. Apply various definitions, results and methods learnt in three theory courses to plot graphs and solve problems.
2. Explore mathematical softwares like Matlab/ Scilab/ Geogebra/ SAGE/ Desmos to solve problems and visualize solids.
3. Test validity of mathematical statements using results and constructing appropriate examples.

Practicals in SIUSMAT41

1. Limits and continuity of scalar fields and vector fields, using definition and otherwise, iterated limits.
2. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
3. Quadric surfaces, gradient, level sets and tangent planes.
4. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
5. Taylor's formula, differentiation of a vector field at a point, finding Hessian/ Jacobian matrix, Mean Value Inequality.
6. Finding maxima, minima and saddle points, second derivative test for extrema of functions of two variables and method of Lagrange multipliers.
7. Miscellaneous Questions based on all units.

Practicals in SIUSMAT42

1. Linear transformation, Kernel, Rank-Nullity Theorem.
2. Linear Isomorphism, Matrix associated with Linear transformations.
3. Inner product and properties, Projection, Orthogonal complements.
4. Orthogonal, orthonormal sets, Gram-Schmidt orthogonalisation
5. Eigenvalues, Eigenvectors, Characteristic polynomial. Applications of Cayley Hamilton Theorem.
6. Diagonalisation of matrix, orthogonal diagonalisation of symmetric matrix and application to quadratic form.
7. Miscellaneous Questions based on all units.

Practicals in SIUSMAT43

1. Finding the general solution of homogeneous and non-homogeneous higher order linear differential equations using method of undetermined coefficients and method of variation of parameters.
2. Solving higher order linear differential equations using inverse operators. Solving Cauchy's equation and Legendre's equation.
3. Solving a system of first order linear ODES having auxiliary equations with real roots.
4. Solving a system of first order linear ODES having auxiliary equations with complex roots.
5. Finding the numerical solution of initial value problems using Taylor's series method, Picard's method, modified Euler's method, Runge-Kutta method of second order and fourth order.
6. Finding the numerical solution of simultaneous ordinary differential equations using fourth order Runge-Kutta method. Finding the numerical solution of two-point linear boundary value problem using the Finite difference method.
7. Miscellaneous theory questions based on all units.

6. Teaching Pattern for semester III & IV

1. Three lectures per week per course in each semester.
2. One practical per week per batch based on all courses. (6 lectures)
3. Minimum 6 practicals to be conducted in each course in each semester.
4. Conduct of Practicals

The Practicals shall be conducted in batches formed as per the University circular. The Practical session shall consist of discussion between the teacher and the students in which students should participate actively. The students should maintain a journal for practicals which should be submitted for checking regularly and at the end of the semester.

7. Scheme of Evaluation for Semesters III & IV

The performance of the learners shall be evaluated in three ways:
(a) Continuous Internal Assessment of 40 marks in each course in each semester.
(b) Semester End Examinations of 60 marks in each course at the end of each semester.
(c) A combined Practical exam of 150 marks for all the three courses at the end of each semester.

(a) Internal Assessment in each Course in each semester

Sr No	Evaluation type	Marks
1	One class test	20
2	Teachers may use various methods to encourage experiential learning and problem-solving skills of the student. Written assignment/ project assignment/ oral or ppt or poster presentation/ reading assignment with viva voce, seminar etc.	20
	Total	40

(b) Semester end examination in each course at the end of each semester (60 marks)

Duration – 2 hours.
Question Paper Pattern: - Four questions each of 15 marks.
One question on each unit (Questions 1, 2, 3).
Question 4 will be based on the entire syllabus.

(c) Practical Examination based on all the three theory courses to be conducted at the end of each semester, total duration 4 hours 30 minutes—

Sr No	Evaluation type	Marks in each course			Total
1	One practical test	40	40	40	120
2	Certified Journal	10	10	10	30
	Total	50	50	50	150

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